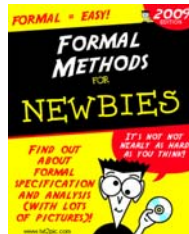


CISC422/853: Formal Methods in Software Engineering: Computer-Aided Verification



Topic 8: Model Checking, Part 2

Juergen Dingel
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Readings:

- Spin book: Chapter 8, pages 178-181 (Search Algorithms)
- Course notes on CTL

Outline

- How to check for
 - **assertion violations & deadlock**
 - Basic DFS
 - **safety properties**
 - expressed as FSAs (in Bogor)
 - expressed as Never Claims (in Spin)
 - expressed as LTL properties (in Spin)
 - **liveness properties**
 - progress labels (in Spin)
 - expressed as Never Claims (in Spin)
 - expressed as LTL properties (in Spin)

Preliminaries

- Let system S be given by n **concurrent threads**
 T_1, \dots, T_n
- Threads T_i execute **asynchronously** in S
- So, A_S , the FSA representing S, is obtained by building the **asynchronous composition** of the A_{T_i} , the FSA representing T_i , that is,

$$A_S = A_{T_1} \parallel \dots \parallel A_{T_n}$$

Check Safety With Assertions

```
checkAssertions( $A_c$ ) {
  seen := { $s_0$ }
  stack := [ $s_0$ ]
  DFS( $s_0$ )
}
```

```
DFS( $s$ ) {
  ws := enabled( $s$ )
  for all a in ws {
    if a=assert( $p$ ) && !eval( $p,s$ ) then
      print("violation",  $s$ +stack)
     $s'$  := execute( $a, s$ )
    if  $s'$  not in seen {
      seen := seen + { $s'$ }
      push( $s'$ , stack)
      DFS( $s'$ )
      pop(stack)
    }
  }
}
```

set of states already explored

states on current path

• get the transitions out of s (possibly "on-the-fly")
• s records state of each thread T_i ,
i.e., $s = (s_{T_1}, \dots, s_{T_n})$

pick one of the transitions to explore

check for assertion violation, if necessary

calculate the successor state

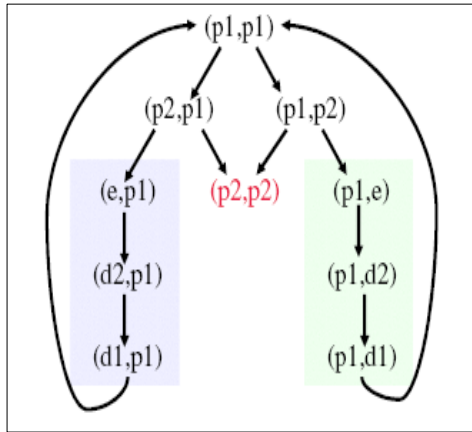
if successor state has been seen before,
ignore it

explore successor state

Check for deadlock is similar!

Check Safety With FSAs: Example

Let's look at an **example system**
 [Phil1 || Phil2]:



```

boolean f1, f2;
thread Phil1() {
  loc pickup1: when !f1 do
    {f1 := true;}
    goto pickup2;
  loc pickup2: when !f2 do
    {f2 := true;}
    goto eating;
  loc eating: do {}
    goto drop2;
  loc drop2: do {f2 := false;}
    goto drop1;
  loc drop1: do {f1 := false;}
    goto pickup1;
}
    
```

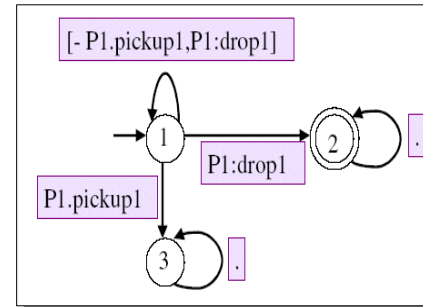
Check Safety With FSAs: Example (Cont'd)

... and an **example property** P_1 :

"Phil1 must pickup Fork1 before dropping it"

$\neg P_1$ as reg. exp.: $[- P1.pickup1, P1.drop1]^*; P1.drop1; .^*$

$\neg P_1$ as a FSA:



$\neg P_1$ as a BIR program:

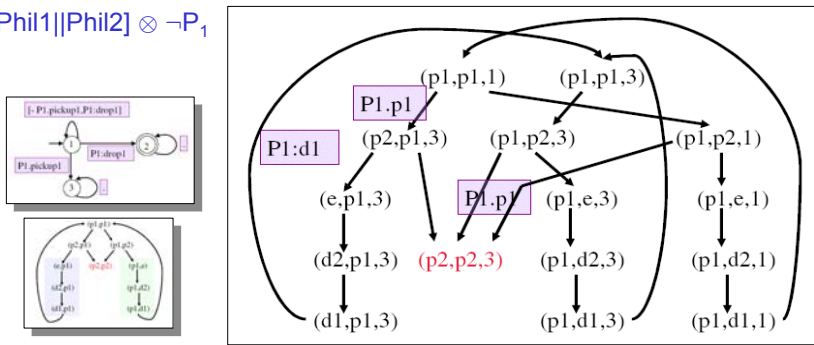
```

extension Property for ...
{expdef boolean transform(string, string);}
function notP1() {
  loc init:
    when Property.transform("Phil1", "drop1")
    do {} goto bad;
    when Property.transform("Phil1", "pickup1")
    do {} goto good;
  loc good do {} goto good;
  loc bad do {} goto bad;
}
    
```

Check Safety With FSAs: Example (Cont'd)

To check whether [Phil1 || Phil2] satisfies P_1 , we take the **synchronous composition** of [Phil1 || Phil2] and $\neg P_1$:

$[Phil1||Phil2] \otimes \neg P_1$



Every word/execution ending in (*, *, 2) is

- in $L([Phil1||Phil2] \otimes \neg P_1)$
- a violating execution!

Here:

- no violating executions
- system satisfies P_1 !

Check Safety With FSAs: Example (Cont'd)

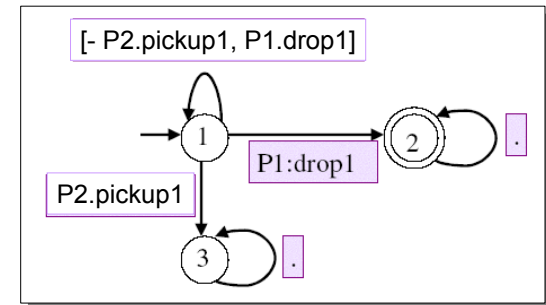
Here's **another property** P_2

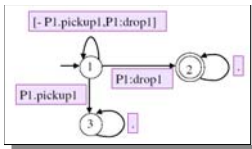
"Phil2 must pickup Fork1 before Phil1 can drop it"

▪ $\neg P_2$ as regular expression:

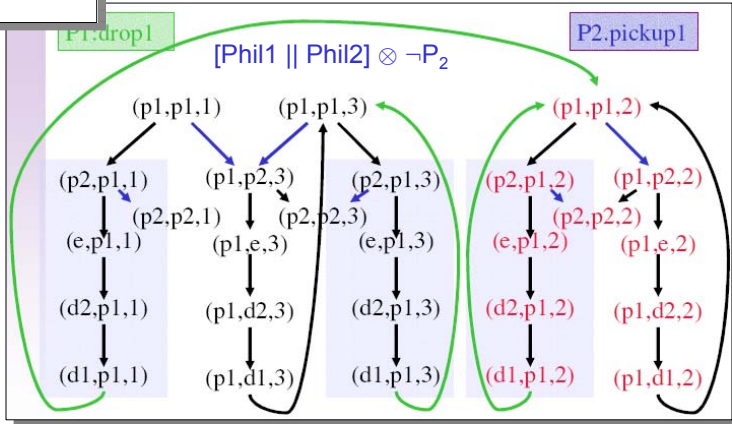
$[- P2.pickup1, P1.drop1]^*; P1.drop1; .^*$

▪ $\neg P_2$ as FSA:





Check Safety With FSAs: Example (Cont'd)



Every word/execution ending in (*, *, 2) is

- in $L([\text{Phil1}||\text{Phil2}] \otimes \neg P_2)$
- a violating execution

Here:

- lots of violating executions
- system does not satisfy P_2

Check Safety With FSAs

- Let $A_S = A_{T_1} || \dots || A_{T_n}$
- Let A_P be FSA expressing safety property P

3 Steps:

1. build FSA $A_{\neg P}$ for negation of P
 - $A_{\neg P}$ must be total (use stutter extension)
2. build $A_S \otimes A_{\neg P}$, synchronous product of A_S and $A_{\neg P}$
3. do basic DFS on $A_S \otimes A_{\neg P}$
 - if a transition puts $A_{\neg P}$ into final state, then
 - Violation! Print contents of DFS stack as error trace
 - $L(A_S \otimes A_{\neg P})$ not empty
 - if $A_{\neg P}$ never reaches a final state, then
 - no violation! S satisfies P
 - $L(A_S \otimes A_{\neg P})$ empty

can be done at the same time ("on-the-fly" model checking, e.g., Spin, Bogor)

Check Safety With FSAs (Cont'd)

- Let A_S be $(S_S, s_{0,S}, L_S, \delta_S, F_S)$
- Let $A_{\neg P}$ be $(S_{\neg P}, s_{0,\neg P}, L_{\neg P}, \delta_{\neg P}, F_{\neg P})$ where $A_{\neg P}$ is deterministic

```

checkSafety( $A_S, A_{\neg P}$ ) {
  seen := {(s0,S, s0,¬P)}
  stack := [(s0,S, s0,¬P)]
  DFS((s0,S, s0,¬P))
}
  
```

update state of $A_{\neg P}$

$A_{\neg P}$ in final state?

```

DFS((s, p)) {
  ws := enabled(s)
  for each a in ws {
    s' := execute(a, s)
    p' := (a in L¬P) ? δ¬P(p,a) : p
    if p' in F¬P then
      print("violation", (s',p')+stack)
    if (s',p') not in seen then {
      seen := seen + {(s',p')}
      push((s',p'), stack)
      DFS((s',p'))
      pop(stack)
    }
  }
}
  
```

new state component for $A_{\neg P}$

Check Safety With Never Claims

- In Spin, safety properties can be expressed using Never Claims
- Never Claims representing safety properties are FSAs
- Let $NC_{\neg P}$ be NC expressing negation of safety property P
- Check as before, except don't need to build negation

2 Steps:

1. build $A_S \otimes NC_{\neg P}$, synchronous product of A_S and $NC_{\neg P}$
2. do basic DFS on $A_S \otimes NC_{\neg P}$
 - final state of NC reached when NC "fully matched"
 - S violates P iff $L(A_S \otimes NC_{\neg P})$ not empty

Check Safety With Never Claims (Cont'd)

- Let A_S be $(S_S, s_{0,S}, L_S, \delta_S, F_S)$
- Let $NC_{\neg P}$ be $(S_{\neg P}, s_{0,\neg P}, L_{\neg P}, \delta_{\neg P}, F_{\neg P})$ where $NC_{\neg P}$ is deterministic

```
checkSafety( $A_S, NC_{\neg P}$ ) {
  seen := {( $s_{0,S}, s_{0,\neg P}$ )}
  stack := [( $s_{0,S}, s_{0,\neg P}$ )]
  DFS(( $s_{0,S}, s_{0,\neg P}$ ))
}
```

$NC_{\neg P}$ fully matched?

```
DFS(( $s, p$ )) {
  ws := enabled( $s$ )
  for each  $a$  in  $ws$  {
     $s'$  := execute( $a, s$ )
     $p'$  := ( $a$  in  $L_{\neg P}$ ) ?  $\delta_{\neg P}(p, a)$  :  $p$ 
    if  $p'$  in  $F_{\neg P}$  then
      print("violation", ( $s', p'$ )+stack)
    if ( $s', p'$ ) not in seen then {
      seen := seen + {( $s', p'$ )}
      push(( $s', p'$ ), stack)
      DFS(( $s', p'$ ))
      pop(stack)}
  }
}
```

Check Safety With LTL

- In Spin, safety properties can also be expressed as LTL properties
- Let P be safety property expressed in LTL
- Checking proceeds as before
- 3 Steps:**
 - build FSA $A_{\neg P}$ for negation of P
 - $A_{\neg P}$ must be total (use stutter extension)
 - build $A_S \otimes A_{\neg P}$, synchronous product of A_S and $A_{\neg P}$
 - do basic DFS on $A_S \otimes A_{\neg P}$
 - as before
 - S violates P iff $L(A_S \otimes A_{\neg P})$ not empty

Check Safety: In A Nutshell

- Let S be system with threads T_1, \dots, T_n
- Let P be safety property
- Steps:**
 - build FSA $A_{\neg P}$ for negation of P
 - build $A_S \otimes A_{\neg P}$, synchronous product of A_S and $A_{\neg P}$ where $A_S = A_{T_1} \parallel \dots \parallel A_{T_n}$, asynchronous composition of the T_i
 - do basic DFS on $A_S \otimes A_{\neg P}$
- Complexity:**
 - $O(R)$ where R is # of reachable states in $A_S \otimes A_{\neg P}$

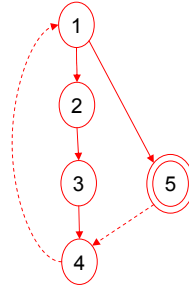
• Spin and Bogor do both steps at the same time ("on-the-fly")
 • SMV carries steps out sequentially (not "on-the-fly")

Check Liveness With Never Claims

- Remember:**
 - NC expresses violation of property
 - NC representing liveness property = Buechi Automaton = FSA + ω -acceptance
- Let**
 - A_S be Buechi automaton representing the system S
 - $NC_{\neg P}$ express violation of liveness property P
 - t be execution of A_S
- Execution t violates P iff**
 - "some 'good thing' never happens along t "
 - iff t in $L^\omega(NC_{\neg P})$
 - iff t causes $NC_{\neg P}$ into an 'acceptance cycle'

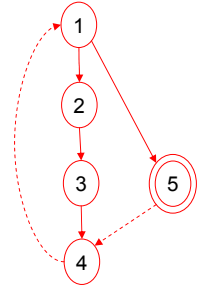
Check Liveness With Never Claims (Cont'd)

- **S satisfies P**
 - iff $L^\omega(A_S \otimes NC_{-P})$ is empty
 - iff $A_S \otimes NC_{-P}$ has no accepting execution
 - iff $A_S \otimes NC_{-P}$ has no execution that ends in an accepting cycle
- **To check if S satisfies P**
 1. build $A_S \otimes NC_{-P}$
 2. check if $A_S \otimes NC_{-P}$ has acceptance cycle
 - How to do that?
 - Basic DFS or BFS is not enough...



Check Liveness With Never Claims (Cont'd)

- **Solution 1:**
 - Compute **strongly connected components** (SCC) in $A_S \otimes NC_{-P}$ (Tarjan's algorithm)
 - $A_S \otimes NC_{-P}$ has acceptance cycle iff
 - $A_S \otimes NC_{-P}$ has SCC such that
 - SCC reachable from initial state, and
 - SCC contains at least one accepting state
- **Solution 2: (easier)**
 - Check if $A_S \otimes NC_{-P}$ has at least one state s s.t.
 - (1) s is accepting
 - (2) s reachable from initial state
 - (3) s is reachable from itself
 - Implementation: **Nested DFS**
 - First DFS to find s s.t. (1) and (2)
 - Then, nested DFS to check (3)



Check Liveness With Never Claims (Cont'd)

- Let A_S be $(S_S, s_{0,S}, L_S, \delta_S, F_S)$
- Let NC_{-P} be $(S_{-P}, s_{0,-P}, L_{-P}, \delta_{-P}, F_{-P})$ where NC_{-P} is deterministic

```

checkLiveness( $A_S, NC_{-P}$ ) {
  seen1 := {( $s_{0,S}, s_{0,-P}$ )}
  stack1 := [( $s_{0,S}, s_{0,-P}$ )]
  DFS(( $s_{0,S}, s_{0,-P}$ ))
}
  
```

```

DFS(( $s, p$ )) {
  ws1 := enabled( $s$ )
  for each  $a$  in ws {
     $s' := execute(a, s)$ 
     $p' := (a \text{ in } L_{-P}) ? \delta_{-P}(p, a) : p$ 
    if ( $s', p'$ ) not in seen then {
      seen1 := seen1 + {( $s', p'$ )}
      push(( $s', p'$ ), stack1)
      DFS(( $s', p'$ ))
    }
    if  $p'$  in  $F_{-P}$  then {
      seen2 = {( $s', p'$ )}
      stack2 = [( $s', p'$ )]
      NDFS(( $s', p'$ ), ( $s', p'$ ))
    }
  }
  pop(stack1)
}
  
```

is p' accepting state?

start nested DFS if p' is an accepting state of NC_{-P}

Check Liveness With Never Claims (Cont'd)

```

DFS(( $s, p$ )) {
  ws1 := enabled( $s$ )
  for each  $a$  in ws {
     $s' := execute(a, s)$ 
     $p' := (a \text{ in } L_{-P}) ? \delta_{-P}(p, a) : p$ 
    if ( $s', p'$ ) not in seen then {
      seen1 := seen1 + {( $s', p'$ )}
      push(( $s', p'$ ), stack1)
      DFS(( $s', p'$ ))
      if  $p'$  in  $F_{-P}$  then {
        seen2 = {( $s', p'$ )}
        stack2 = [( $s', p'$ )]
        NDFS(( $s', p'$ ), ( $s', p'$ ))
      }
    }
  }
  pop(stack1)
}

NDFS(( $s, p$ ), start) {
  ws2 := enabled( $s$ )
  for each  $a$  in ws2 {
     $s' := execute(a, s)$ 
     $p' := (a \text{ in } L_{-P}) ? \delta_{-P}(p, a) : p$ 
    if ( $s', p'$ ) = start then
      print("violation", stack1+stack2)
    if ( $s', p'$ ) not in seen2 then {
      seen2 := seen2 + {( $s', p'$ )}
      push(( $s', p'$ ), stack2)
      NDFS(( $s', p'$ ), start)
      pop(stack2)
    }
  }
}
  
```

acceptance cycle found!

Check Liveness With Never Claims (Cont'd)

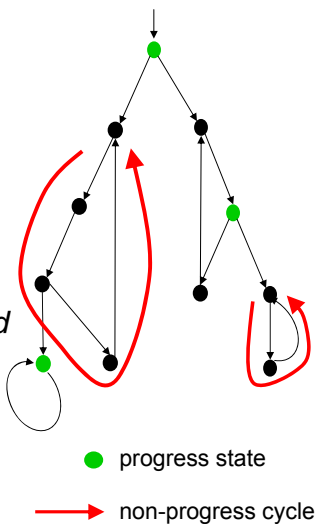
- Let $NC_{\neg P}$ be NC expressing negation of liveness property P
- **2 Steps:**
 1. build $A_S \otimes NC_{\neg P}$, synchronous product of A_S and $NC_{\neg P}$
 2. do nested DFS on $A_S \otimes NC_{\neg P}$ to search for acceptance cycle
 - S violates P iff
 - $A_S \otimes NC_{\neg P}$ has acceptance cycle
 - $L^\omega(A_S \otimes NC_{\neg P})$ not empty

Check Liveness With LTL

- Let P be an LTL formula expressing a liveness property
- Build $NC_{\neg P}$ representing negation of P
- Then, as before
- **3 Steps:**
 1. build $NC_{\neg P}$, never claim representing $\neg P$
 2. build $A_S \otimes NC_{\neg P}$, synchronous product of A_S and $NC_{\neg P}$
 3. do nested DFS on $A_S \otimes NC_{\neg P}$ to search for acceptance cycle
 - S violates P iff
 - $A_S \otimes NC_{\neg P}$ has acceptance cycle
 - $L^\omega(A_S \otimes NC_{\neg P})$ not empty

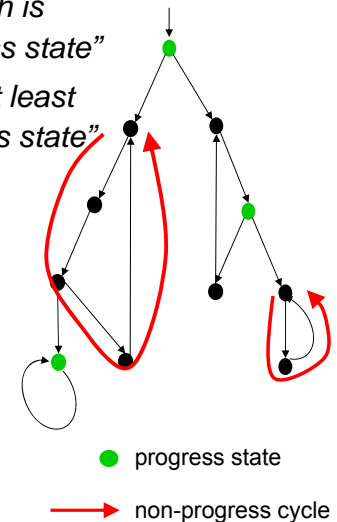
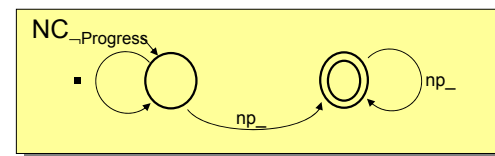
Check Liveness With Progress Labels

- **Need to find non-progress cycles**
- **Remember:** In Spin,
 - whether or not the system makes progress in a given state s is observable
 - $np_$ false in s iff at least one process is at progress label in s
- Let **Progress** be “every state along every path is always eventually followed by a progress state”
- **Idea:** Use $np_$ to express *Progress* and \neg *Progress* as LTL formulas
- **Which?**



Check Liveness With Progress Labels (Cont'd)

- **Progress** = “every state along every path is always eventually followed by a progress state”
- \neg **Progress** = “at least one state along at least one path is never followed by a progress state”
- In LTL using $np_$:
 - **Progress** = $\Box \langle \rangle \neg np_$
 - \neg **Progress** = $\langle \rangle \Box np_$
- As Never Claim $NC_{\neg \text{Progress}}$:

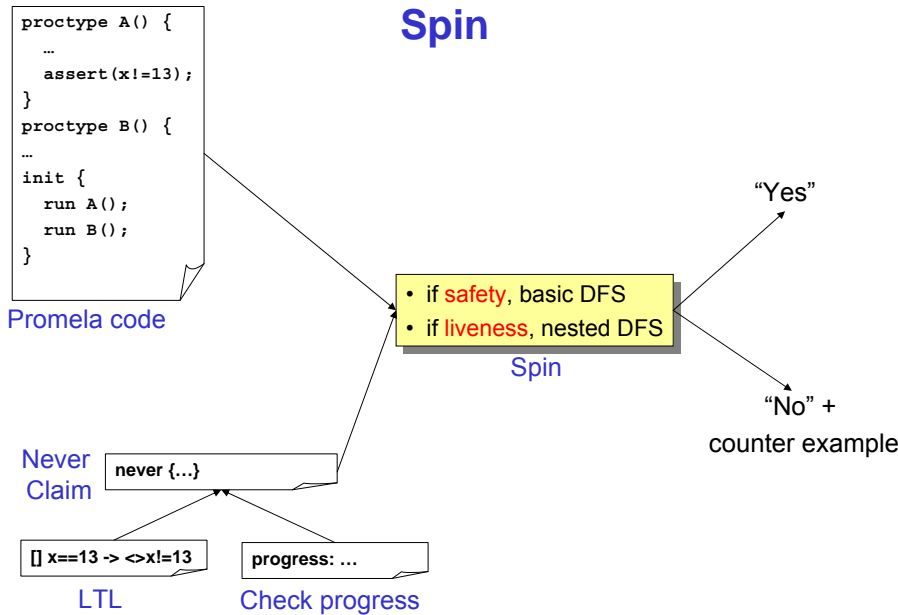


Check Liveness With Progress Labels (Cont'd)

- 3 Steps:
 - build $NC_{\neg\text{Progress}}$, never claim representing non-progress
 - build $A_S \otimes NC_{\neg\text{Progress}}$, synchronous product of A_S and $NC_{\neg\text{Progress}}$
 - do nested DFS on $A_S \otimes NC_{\neg\text{Progress}}$ to search for non-progress cycle
 - S violates Progress iff
 - $A_S \otimes NC_{\neg\text{P}}$ has acceptance cycle
 - $L^\omega(A_S \otimes NC_{\neg\text{P}})$ not empty

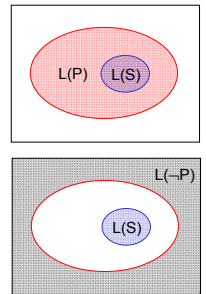
Check ~~Safety~~ Liveness: In A Nutshell

- Let S be system with threads T_1, \dots, T_n
- Let P be ~~safety~~ ^{liveness} property
- Steps: ^{Buechi Automaton}
 - build ~~FSA~~ $A_{\neg P}$ for negation of P
 - build $A_S \otimes A_{\neg P}$, synchronous product of A_S and $A_{\neg P}$ where $A_S = A_{T_1} \parallel \dots \parallel A_{T_n}$, asynchronous composition of the T_i
 - do ~~basic~~ ^{nested} DFS on $A_S \otimes A_{\neg P}$
 - Spin and Bogor do both steps at same time ("on-the-fly")
 - SMV carries steps out sequentially
- Complexity:
 - ~~$O(R)$~~ ^{$O(2 \cdot R)$} where R is # of reachable states in $A_S \otimes A_{\neg P}$



The Language-Theoretic View

- $L(S)$: system executions
- $L(P)$: executions satisfying the property
- Need to determine: $L(S) \subseteq L(P)$
- Observation: $A \subseteq B$ iff $(A \cap \neg B) = \emptyset$
- So, to see if $L(S) \subseteq L(P)$, we
 - Step 1: take $\neg P$
 - Step 2: see if $L(S) \cap L(\neg P)$ is empty, that is, if there does not exist an execution t such that
 - S can do t, that is, t in $L(S)$, and
 - t violates P, that is, t is in $L(\neg P)$
 - Step 2 will succeed precisely when $S \otimes \neg P$ has no accepting executions
- Theorem: Buechi Automata are closed under negation, union and intersection



Important Properties of Buechi-Automata

Buechi automata are closed under complement, union, and intersection.

Let A and B be Buechi-automata. Then,

- $\neg A$ denotes the automaton that accepts precisely the words not accepted by A:

$$L^\omega(\neg A) = \{w \mid w \notin L^\omega(A)\}$$

- $A \cup B$ denotes the automaton that accepts precisely the words accepted by A or by B:

$$L^\omega(A \cup B) = L^\omega(A) \cup L^\omega(B)$$

- Similarly for $A \cap B$

Weak Fairness

- So far, it's possible that along a counter example a process stops moving although it is enabled
 \Rightarrow such counter examples are not very realistic (why?)

Definition: An ω -run σ satisfies the *weak fairness* requirement if it contains infinitely many transitions from every process (component automaton in the asynchronous product) that is *enabled* (has an executable action) *infinitely long* in σ

- Nested DFS can be adapted to enforce fairness (more details in Spin book Chapter 8)
- Cost: linear increase in complexity (in # of processes)

Complexity and Optimization

- Size of $A_S \otimes A_{-P}$

- $R = \#$ of reachable states in $A_S \otimes A_{-P}$
- $R = R_S \cdot R_{-P}$ where
 - $R_S = \#$ of reachable states in A_S (typically: $10^9 \dots 10^{11}$)
 - $R_{-P} = \#$ of reachable states in A_{-P} (typically: 1..4)

- Size of A_S

- $R_S = R_{T1} \cdot \dots \cdot R_{Tn} \sim R_T^n$

- Size of T

- $R_T = (\# \text{ loc's in } T) \cdot |\text{dtype}_1| \cdot \dots \cdot |\text{dtype}_m| \sim (\# \text{ loc's in } T) \cdot |\text{dtype}|^m$

- Thus,

- $R_S = (\# \text{ loc's in } T) \cdot |\text{dtype}|^m)^n$

R_S increases with

- # of processes n (exponentially)
- # of variables m
- size of data types
- size of process

Complexity and Optimization (Cont'd)

- Size of $A_S \otimes A_{-P}$

- $R = R_S \cdot R_{-P} = (\# \text{ loc's in } T) \cdot |\text{dtype}|^m)^n \cdot R_{-P}$

- Reduce R by

reducing

- # of processes n (exponentially)
- # of variables m
- size of data type dtype
- size of process T
- size of specification P

user

using

- partial order reduction
- statement merging
- abstraction

checker/user

- Reduce memory requirement by

- compression

CTL Model Checking Algorithm (1)

- So much for LTL model checking
- Now, on to CTL model checking
- Algorithm quite different, because CTL quite different from LTL

CTL Model Checking Algorithm (2)

Definition: A set of connectives S is **adequate** for CTL iff for every CTL formula φ , there exists an equivalent CTL formula $T(\varphi)$ that only contains the connectives in S

Theorem: $\{\neg, \vee, EX, AF, EU\}$ is adequate for CTL

Proof:

$\varphi_1 \wedge \varphi_2$	\leftrightarrow	$\neg(\neg\varphi_1 \vee \neg\varphi_2)$
$\varphi_1 \rightarrow \varphi_2$	\leftrightarrow	$\neg\varphi_1 \vee \varphi_2$
$AX\varphi$	\leftrightarrow	$\neg EX\neg\varphi$
$AG\varphi$	\leftrightarrow	$\neg EF\neg\varphi$
$EG\varphi$	\leftrightarrow	$\neg AF\neg\varphi$
$EF\varphi$	\leftrightarrow	$E[\text{tt} \cup \varphi]$
$A[\varphi_1 \cup \varphi_2]$	\leftrightarrow	$\neg(EG\neg\varphi_2 \vee E[\neg\varphi_2 \cup \neg\varphi_1 \wedge \varphi_2])$
	\leftrightarrow	$AF\varphi_2 \wedge \neg E[\neg\varphi_2 \cup \neg\varphi_1 \wedge \varphi_2]$

CTL Model Checking Algorithm (3)

Recall

▪ $AG\varphi$	\leftrightarrow	$\varphi \wedge AX AG\varphi$
▪ $EG\varphi$	\leftrightarrow	$\varphi \wedge EX EG\varphi$
▪ $AF\varphi$	\leftrightarrow	$\varphi \vee AX AF\varphi$
▪ $EF\varphi$	\leftrightarrow	$\varphi \vee EX EF\varphi$
▪ $A[\varphi_1 \cup \varphi_2]$	\leftrightarrow	$\varphi_2 \vee (\varphi_1 \wedge AX A[\varphi_1 \cup \varphi_2])$
▪ $E[\varphi_1 \cup \varphi_2]$	\leftrightarrow	$\varphi_2 \vee (\varphi_1 \wedge EX E[\varphi_1 \cup \varphi_2])$

CTL Model Checking Algorithm (4)

Input: FSM $M=(S, s_0, L, \rightarrow, F)$ and CTL formula φ over AP

Output: “yes” if $M \models \varphi$, “no” otherwise

Step 0: Let φ' be $T(\varphi)$

Step 1: For all subformulas ψ in φ' (starting w/ smallest)

including φ' , label all states s in M satisfying ψ :

Sat(ψ) = CASE ψ OF

$p \in AP$: label a state s w/ p if p true in s

$\neg\psi'$: $Sat(\psi')$; label a state s w/ $\neg\psi$ if s is not labeled w/ ψ

$\psi_1 \vee \psi_2$: $Sat(\psi_1)$; $Sat(\psi_2)$; label a state s w/ $\psi_1 \vee \psi_2$ if s labeled w/ ψ_1 or ψ_2

$EX \psi'$: $Sat(\psi')$; label a state s w/ $EX\psi'$ if at least one successor of s is labeled w/ ψ'

$AF\psi'$: $Sat(\psi')$;

Repeat

label state s w/ $AF\psi'$ if s labeled w/ ψ' or all successors of s labeled w/ $AF\psi'$

Until labeling doesn't change anymore, i.e., a “fixed point” is reached

$E[\psi_1 \cup \psi_2]$: $Sat(\psi_1)$; $Sat(\psi_2)$;

Repeat

label state s w/ $E[\psi_1 \cup \psi_2]$ if s labeled w/ ψ_2 or (s labeled w/ ψ_1 and at least one successor of s labeled w/ $E[\psi_1 \cup \psi_2]$)

Until labeling doesn't change anymore, i.e., a “fixed point” is reached

CTL Model Checking Algorithm (5)

Input: FSM $M=(S, s_0, L, \rightarrow, F)$ and CTL formula φ over AP

Output: “yes” if $M\models\varphi$, “no” otherwise

Step 0: Let φ' be $T(\varphi)$

Step 1: For all subformulas ψ in φ' (starting w/ smallest) including φ' , label all states s in M satisfying ψ :

Step 2: If s_0 labeled with φ , then output “yes”, else output “no”

Example:

$M\models AF p?$

“yes”

Complexity: $O((|S|+|\rightarrow|) \cdot |\varphi|)$

LTL Model Checking vs. CTL Model Checking

To check $M \models \varphi$

LTL model checking:

- 1) Check if $L(M \otimes A_{\neg\varphi}) = \emptyset$ where $A_{\neg\varphi}$ is non-deterministic Buchi Automaton representing φ
- 2) Check implemented by
 - a) for safety: **DFS** or **BFS**
 - b) for liveness: **nested DFS**
- 3) Note
 - a) execution sequences are **linear** (non-branching)
 - b) transition relation of M **can be computed “on-the-fly”**
 - c) In worst case, $|A_{\neg\varphi}|$ exponential in $|\varphi|$
- 4) Complexity: $O(|M| \cdot 2^{|\varphi|})$, but $|M|$ dominates $2^{|\varphi|}$ in practice
- 5) Sample tools: **Spin, Bogor, JPF**

CTL model checking:

- 1) Check if $(M, s) \models \varphi$ for all $s_0 \in M.S_0$
- 2) Check implemented by
 - a) express φ in terms of $\{\neg, \vee, EX, AF, EU\}$
 - b) **labeling algorithm** $Sat(\varphi)$ that is inductive over structure of φ and uses fixed point computation
- 3) Note
 - a) execution sequences are **branching**
 - b) transition relation of M **cannot be computed on-the-fly**
- 4) Complexity: $O(|M| \cdot |\varphi|)$
- 5) Sample tools: **SMV**

Projects and Presentations

Schedule

- Now: pick project
- Till week of April 6: work on project
- Week of April 6: presentations & summary papers

Presentations

- 20 mins
- group members take turns

Summary papers

- b/w 2-5 pages in ACM SIG Proceedings format
- to be distributed at presentation time